Effect of Wall Flexibility on Dynamic Response of Concrete Rectangular Liquid Storage Tanks under Horizontal and Vertical Ground Motions

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Abstract: In this study, the finite-element method is used to investigate the seismic behavior of rectangular liquid tanks in two-dimensional space. This method is capable of considering both impulsive and convective responses of liquid-tank system. Two different finite-element models corresponding with shallow and tall tank configurations are studied under the effects of both horizontal and vertical ground motions using the scaled earthquake components of the 1940 El-Centro earthquake record. The containers are assumed fixed to the rigid ground. Fluid-structure interaction effects on the dynamic response of fluid containers are taken into account incorporating wall flexibility. The results show that the wall flexibility and fluid damping properties have a major effect on seismic behavior of liquid tanks and should be considered in design criteria of tanks. The effect of vertical acceleration on the dynamic response of the liquid tanks is found to be less significant when horizontal and vertical ground motions are considered together. The results in this study are verified and compared with those obtained by numerical methods and other available methods in the literature.

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Introduction

Liquid storage tanks are critical lifeline structures which have become widespread during the recent decades. These structures are extensively used in water supply facilities, oil and gas industries, and nuclear plants for storage of a variety of liquid or liquidlike materials such as oil, liquefied natural gas, chemical fluids, and wastes of different forms. These tanks are exposed to a wide range of seismic hazards and interaction with other sectors of built environment. Heavy damages have been reported due to strong earthquakes such as Niigata in 1964, Alaska in 1964, Parkfield in 1966, Imperial County in 1979, Coalinga in 1983, Northridge in 1994, and Kocaeli in 1999 some of which are reported by Haroun and Ellaithy (1985), Rai (2002), and Sezen and Whittaker (2006).

Problems associated with liquid tanks involve many fundamental parameters. In fact, the dynamic behavior of liquid tanks is governed by the interaction between fluid and structure as well as soil and structure along their boundaries. On the other hand, structural flexibility, fluid properties, and soil characteristics are the factors which are of great importance in analyzing the tank behavior. It has been found that hydrodynamic pressure in a flexible tank can be significantly higher than the corresponding rigid container due to the interaction effects between flexible structure and contained liquid.

Even though there have been numerous studies done on the fluid-structure interaction effects in liquid containers, most of them are concerned with cylindrical tanks. Housner (1963) developed the most commonly used analytical model in which hydrodynamic pressure induced by seismic excitations is separated into impulsive and convective components using lumped mass approximation. The fluid was assumed incompressible, inviscid, and the structure was assumed to be rigid. This model has been adopted with some modifications in most of the current codes and standards.

Yang (1976) studied the effects of wall flexibility on the pressure distribution in liquid and corresponding forces in the tank structure through an analytical method using a single degree of freedom (DOF) system with different modes of vibrations. Also, Veletsos and Yang (1977) developed flexible anchored tank linear models and found that the pressure distribution for the impulsive mode of rigid and flexible tanks were similar, but also discovered that the magnitude of the pressure was highly dependent on the wall flexibility.

Minowa (1980, 1984) investigated the effect of flexibility of tank walls and hydrodynamic pressure acting on the wall. Also, experimental studies were carried out to determine the dynamic characteristics of rectangular tanks.

Haroun (1984) presented a very detailed analytical method in the typical system of loading in rectangular tanks. The hydrodynamic pressure was estimated using classical potential flow approach. The boundary condition was treated as rigid walls. In addition, Haroun (1983) carried out a series of experiments including ambient and forced vibration tests. Three full scale water storage tanks were tested to determine the natural frequencies and mode shapes of vibrations. Also, Haroun and Tayel (1985) used the finite-element method (FEM) for analyzing dynamic response

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of liquid tanks subjected to vertical seismic ground motions.

Veletsos and Tang (1986) analyzed liquid storage tanks subjected to vertical ground motion on both rigid and flexible supporting media. Haroun and Abou-Izzeddine (1992) conducted a parametric study of numerous factors affecting the seismic soil tank interaction under vertical excitations.

Veletsos et al. (1992) presented a refined method for evaluating the impulsive and convective components of response of liquid-storage tanks. They found that the convective components of response are insensitive to the flexibilities of the tank wall and supporting soils, and may be computed considering both the tank and the supporting medium to be rigid.

Kim et al. (1996) further developed analytical solution methods and presented the response of filled flexible rectangular tanks under vertical excitation. Their method is simple and convenient for practical purpose but the flexibility of wall was not thoroughly considered. Park et al. (1992) performed research studies on dynamic response of the rectangular tanks. They used the boundary element method (BEM) to obtain hydrodynamic pressure distribution and FEM to analyze the solid wall.

Subhash Babu and Bhattacharyya (1996) developed a numerical scheme using finite-element technique to calculate the sloshing displacement of liquid and pressure developed to such sloshing. Koh et al. (1998) presented a coupled BEM-FEM, including free sloshing motion, to analyze three-dimensional rectangular storage tanks subjected to horizontal ground motion. In this study, the tank structure was modeled using the FEM and the fluid domain using the indirect BEM.

Dogangun et al. (1997) investigated the seismic response of liquid-filled rectangular storage tanks using analytical methods, and the FEM implemented in the general purpose structural analysis computer code SAPIV. The liquid was assumed to be linear-elastic, inviscid, and compressible. A displacement-based fluid finite-element was employed to allow for the effects of the liquid. The effectiveness of the Lagrangian approach for the seismic design of tanks and the effects of wall flexibility on their dynamic behavior were investigated.

Chen and Kianoush (2005) used the sequential method to calculate hydrodynamic pressure in two-dimensional (2D) rectangular tanks including wall flexibility effects. However, fluid sloshing of liquid was ignored in their study. Also, Kianoush and Chen (2006) investigated the dynamic behavior of rectangular tanks subjected to vertical seismic vibrations in a 2D space. The importance of vertical component of earthquake on the overall response of tank-fluid system was discussed. In addition, Kianoush et al. (2006) introduced a new method for seismic analysis of rectangular containers in 2D space in which the effects of both impulsive and convective components are accounted for in time domain.

Livaoglu (2008) evaluated the dynamic behavior of fluid-rectangular tank-foundation system with a simple seismic analysis procedure. In this procedure, interaction effects were presented by Housner's two mass approximations for fluid and the cone model for soil foundation system.

In this study, a comprehensive investigation of dynamic behavior of concrete rectangular tanks is carried out using the FEM in 2D space in which the coupled fluid-structure equations are solved using direct integral method. Effects of wall flexibility, damping properties of liquid, and sloshing motion are taken into account. Also, both horizontal and vertical components of an earthquake are applied in the procedure. This study is part of an ongoing research which is being conducted at Ryerson University. This study has led to some new findings which are presented with the aid of two different tank configurations that are analyzed under time-history excitations. One of the major advantages of proposed method is in accounting for fluid damping properties and considering impulsive and convective components separately which have not been considered in previous studies. In addition, the effect of vertical acceleration has been investigated in detail. The convective and impulsive pressure distributions as well as time-history sloshing of liquid are discussed. Also, a detailed comparison with current practice is presented in this paper.

Analysis Method for the Tank-Fluid System

In liquid domain, the hydrodynamic pressure distribution is governed by the pressure wave equation. Because of the small volume of containers, the velocity of pressure wave assumed to be infinity. Assuming that water is incompressible and neglecting its viscosity, the small-amplitude irrotational motion of water is governed by the 2D wave equation

$$\nabla^2 P(x,y,t) = 0 \tag{1}$$

where $P(x,y,t)$=hydrodynamic pressure in excess of hydrostatic pressure.

The hydrodynamic pressure in Eq. (1) is due to the horizontal and vertical seismic excitations of the walls and bottom of the container. The motion of these boundaries is related to hydrodynamic pressure by boundary conditions. For earthquake excitation, the appropriate boundary condition at the interface of liquid and tank is governed by

$$\frac{\partial P(x,y,t)}{\partial n} = -\rho a_n(x,y,t) \tag{2}$$

where ρ=density of liquid and a_n=component of acceleration on the boundary along the direction outward normal n. No wave absorption is considered in the interface boundary condition.

It is clear that for rigid wall boundary condition the component of acceleration on the right side of Eq. (2) is equal to ground acceleration whereas, for flexible boundary condition this term is equal to ground acceleration plus relative acceleration of the flexible wall.

Accounting for the small-amplitude gravity waves on the free surface of the liquid, the resulting boundary condition is given as

$$\frac{1}{g}\frac{\partial^2 P}{\partial t^2} + \frac{\partial P}{\partial y} = 0 \tag{3}$$

In which y is the vertical direction and g is the gravitational acceleration.

Applying the small-amplitude wave boundary condition will lead to an evaluation of convective pressure distribution in the liquid domain which is of great importance in liquid containers. However, due to the large amplitude of sloshing under the strong seismic excitations and turbulence effects in liquid tanks, more complicated boundary conditions on the surface of liquid are needed to accurately model the convective motions such as works done by Chen et al. (1996). This is especially the case in shallow tank models which may not completely follow the linearized boundary condition equations. In a recent study done by Virella et al. (2008), the influence of nonlinear wave theory on the sloshing natural periods and their modal pressure distribution for rectangular tanks with H_L/L_x ratios ranging from 0.4 to 1.65 was investigated. They concluded that the nonlinearity of the surface wave does not have a major effect on the pressure distribution on the walls and on natural sloshing frequencies. In the present study, two different tank configurations namely a shallow and a tall tank are analyzed as will be discussed later. The ratios of H_L/L_x are 0.37 and 1.26 for the shallow and tall tanks, respectively. In this case, the linearized boundary condition is appropriate particularly for practical applications. Neglecting the gravity wave effects leads to the free surface boundary condition which is appropriate for impulsive motion of liquid. The related governing equation is given as

$$P(x,H_l,t)=0 \tag{4}$$

where H_l=height of liquid in the container.

Using finite-element discretization and discretized formulation of Eq. (1), the wave equation can be written as the following matrix form:

$$[G]\{\ddot{\mathbf{P}}\}+[H]\{\mathbf{P}\}=\{\mathbf{F}\} \tag{5}$$

In which $G_{i,j}=\Sigma G_{i,j}^e$, $H_{i,j}=\Sigma H_{i,j}^e$, and $F_i=\Sigma F_i^e$. The coefficients $G_{i,j}^e$, $H_{i,j}^e$, and F_i^e for an individual element are determined using the following expressions:

$$G_{i,j}^e=\frac{1}{g}\int_{l_e} N_iN_j dl$$

$$H_{i,j}^e=\int_{A_e}\left(\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x}+\frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y}\right)dA$$

$$\{\mathbf{F}\}=\{\mathbf{F}_i\}-\rho[Q]^T(\{\ddot{\mathbf{U}}\}+\{\ddot{\mathbf{U}}_g\})$$

$$F_i^e=\int_{l_e} N_i\frac{\partial P}{\partial n}dl \tag{6}$$

where N_i=shape function of the ith node in the liquid element; $\{\ddot{\mathbf{U}}\}$=acceleration vector of nodes in the structure domain; $\{\ddot{\mathbf{U}}_g\}$=ground acceleration vector applied to the system; and $[Q]$=coupling matrix. l_e and A_e are the integration over side and area of the element, respectively.

In the above formulation, matrices $[H]$ and $[G]$ are constants during the analysis while the force vector $\{\mathbf{F}\}$, pressure vector $\{\mathbf{P}\}$, and its derivatives are the variable quantities. In the coupling system of liquid-structure the pressures are applied to the structure surface as the loads on the container walls. The general equation of fluid-structure can be written in the following form:

$$[M]\{\ddot{\mathbf{U}}\}+[C]\{\dot{\mathbf{U}}\}+[K]\{\mathbf{U}\}=\{f_1\}-[M]\{\ddot{\mathbf{U}}_g\}+[Q]\{\mathbf{P}\}=\{\mathbf{F}_1\}+[Q]\times\{\mathbf{P}\}$$

$$[G]\{\ddot{\mathbf{P}}\}+[C']\{\dot{\mathbf{P}}\}+[H]\{\mathbf{P}\}=\{\mathbf{F}\}-\rho[Q]^T(\{\ddot{\mathbf{U}}\}+\{\ddot{\mathbf{U}}_g\})=\{\mathbf{F}_2\}-\rho[Q]^T\{\ddot{\mathbf{U}}\} \tag{7}$$

where $[M]$, $[C]$, and $[K]$=mass, damping and stiffness matrices of structure. The term $[C']$ is the matrix representing the damping of liquid which is dependent on the viscosity of liquid and wave absorption in liquid domain and boundaries and is rigorously determined. As previously discussed, the matrix $[Q]$ transfers the liquid pressure to the structure as well as structural acceleration to the liquid domain.

With two-node interface elements with x and y transitional DOF at each node on the face of the container wall, and corresponding two-node interface elements with pressure DOF at each node attached to the liquid elements, the coupling matrix is given as

$$[Q]^e=\int_{l_e}\begin{bmatrix}\alpha_1 N_1^s N_1^f & \alpha_1 N_1^s N_2^f\\ \beta_1 N_1^s N_1^f & \beta_1 N_1^s N_2^f\\ \alpha_1 N_2^s N_1^f & \alpha_1 N_2^s N_2^f\\ \beta_1 N_2^s N_1^f & \beta_1 N_2^s N_2^f\end{bmatrix} dl_e \tag{8}$$

where N^f and N^s=shape function in fluid and the structure domain, respectively. Also, α_1 and β_1 are the direction cosines of the nodes of the surface element on the wet face of the structure.

The direct integration scheme is used to find the displacement and hydrodynamic pressure at the end of time increment $i+1$ given the displacement and hydrodynamic pressure at i. The Newmark-β method is used for discretization of both equations (implicit-implicit method). In this method $\{\dot{U}\}_{i+1}$, $\{U\}_{i+1}$, $\{\dot{P}\}_{i+1}$, and $\{P\}_{i+1}$ can be written as follows:

$$\{\dot{U}\}_{i+1}=\{\dot{U}\}_{i+1}^p+\gamma\Delta t\{\ddot{U}\}_{i+1}$$

$$\{\dot{U}\}_{i+1}^p=\{\dot{U}\}_i+(1+\gamma)\Delta t\{\ddot{U}\}_i$$

$$\{U\}_{i+1}=\{U\}_{i+1}^p+\beta\Delta t^2\{\ddot{U}\}_{i+1}$$

$$\{U\}_{i+1}^p=\{U\}_i+\Delta t\{\dot{U}\}_i+(0.5-\beta)\Delta t^2\{\ddot{U}\}_i$$

$$\{\dot{P}\}_{i+1}=\{\dot{P}\}_{i+1}^p+\gamma\Delta t\{\ddot{P}\}_{i+1}$$

$$\{\dot{P}\}_{i+1}^p=\{\dot{P}\}_i+(1-\gamma)\Delta t\{\ddot{P}\}_i$$

$$\{P\}_{i+1}=\{P\}_{i+1}^p+\beta\Delta t^2\{\ddot{P}\}_{i+1}$$

$$\{P\}_{i+1}^p=\{P\}_i+\Delta t\{\dot{P}\}_i+(0.5-\beta)\Delta t^2\{\ddot{P}\}_i \tag{9}$$

where γ and β=integration parameters. Further descriptions regarding the direct integration method can be found in the studies done by Mirzabozorg et al. (2003).

Damping Characteristics of Liquid Sloshing

Under free oscillations, the motion of free liquid surface decays due to damping forces created by viscous boundary layers. Basically, the damping factor depends on the liquid height, liquid kinematic velocity, and tank dimensions. From this point of view, evaluation of damping characteristic for a fluid-tank system needs more considerations. However, due to lack of sufficient data in this field, the classical damping scheme is used in the finite-element model. Considering impulsive and convective parts of liquid domain, damping matrix can be given as

$$\lfloor C_f \rfloor=a[G]+b[H] \tag{10}$$

In which a and b are computed by Rayleigh damping method. In this equation, coefficient a is calculated based on fundamental frequency of liquid sloshing to present the convective part of the response while coefficient b is computed based on fundamental frequency of the tank which is related to the impulsive term. According to ACI 350.3-06 [American Concrete Institute (ACI) Committee 350 2006], for a given rectangular prismatic tank

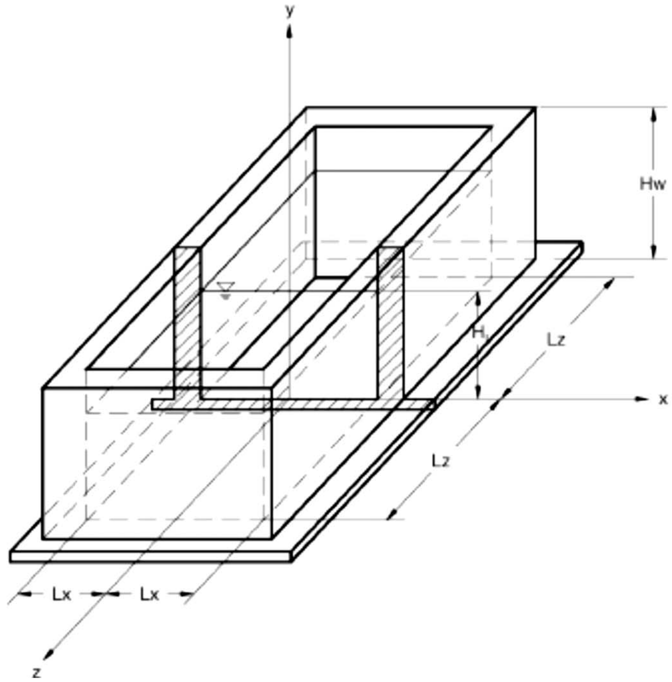


Fig. 1. Schematic configuration of rectangular liquid tank

shown in Fig. 1, the fundamental period and frequency of liquid depending on fill depth are given by

$$f_{\omega} = \frac{1}{2\pi}\sqrt{(n\pi g/a)\tanh(n\pi H_L/a)}$$

$$T_{\omega} = \frac{1}{f_{\omega}} \tag{11}$$

In which $a = 2L_x$ and H_L is the height of the liquid. Parameter n is associated with mode number.

Sloshing in a tank without any antisloshing device is usually damped by viscous forces. One more complete investigations which have been done by Mikishev and Dorozhkin (1961) showed that in a storage tank with rational dimensions viscous damping is less than 0.5%.

In the proposed FE procedure, Rayleigh damping as mentioned previously is used in the direct step-by-step integration method. The stiffness proportional damping equivalent to 5% of critical damping is assumed as structural damping for concrete material. For sloshing and impulsive behaviors of water 0.5 and 5% of critical damping are applied, respectively. These values are chosen as conservative damping ratios based on studies done by Veletsos and Tang (1986) and Veletsos and Shivakumar (1997).

Finite-Element Implementation

In this study, a four-node isoparametric element with two translational degrees of freedom in each node is used in the finite-element procedure to model the tank walls and the base slab. The liquid domain is modeled using four-node isoparametric fluid elements with pressure DOF in each node. Two different model configurations associated with shallow and tall tanks is investigated in 2D space. The FE model configurations for both shallow and tall tanks are shown in Fig. 2.

These tanks have also been used in some previous investigations by Kianoush and Chen (2006), Chen and Kianoush (2005), and Kim et al. (1996). The dimensions and properties of shallow and tall tank are as follows:

- shallow tank

$$\rho_w = 2{,}300 \text{ kg/m}^3 \quad \rho_l = 1{,}000 \text{ kg/m}^3 \quad E_c = 26.44 \text{ GPa} \times \nu = 0.17$$

$$L_x = 15 \text{ m} \quad L_z = 30 \text{ m} \quad H_w = 6.0 \text{ m} \quad H_l = 5.5 \text{ m} \times t_w = 0.6 \text{ m}$$

- tall tank

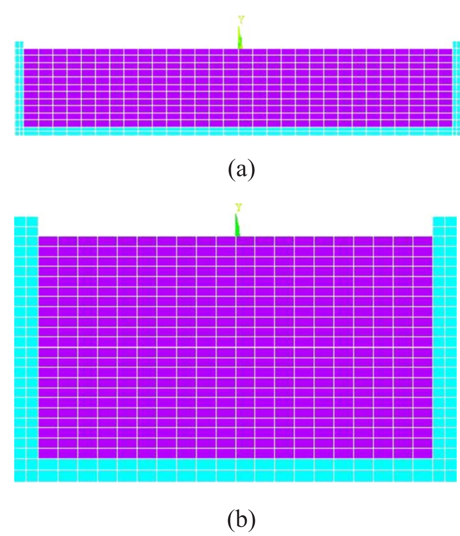


Fig. 2. 2D finite-element model of rectangular liquid tanks considered in this study: (a) shallow tank model; (b) tall tank model

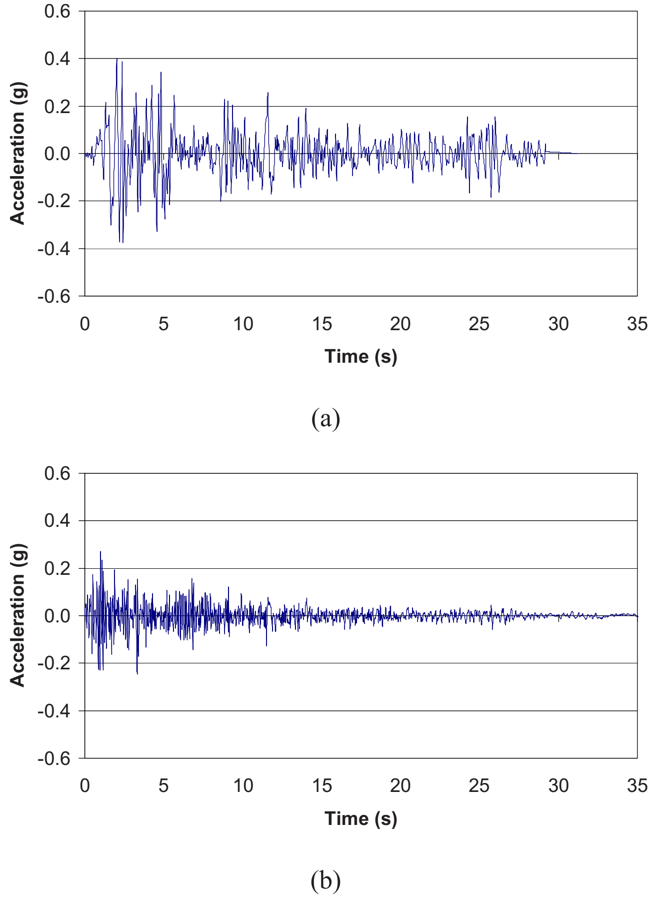


Fig. 3. Scaled components of 1940 El-Centro earthquake: (a) horizontal component; (b) vertical component

$$\rho_w = 2{,}300 \text{ kg/m}^3 \quad \rho_l = 1{,}000 \text{ kg/m}^3 \quad E_c = 20.77 \text{ GPa} \quad \times \nu = 0.17$$

$$L_x = 9.8 \text{ m} \quad L_z = 28 \text{ m} \quad H_w = 12.3 \text{ m} \quad H_l = 11.2 \text{ m} \quad \times t_w = 1.2 \text{ m}$$

A 1-m strip of the tank at the middle of longer dimension is modeled to simulate the 2D behavior of the system. It is assumed that the tank is supported on a rigid foundation and soil-structure effects can be neglected. It is also assumed that the tank is anchored at its base and the effects of uplift pressure are not considered.

The horizontal and vertical components recorded for 1940 El-Centro earthquake are used as excitations of the tank-liquid system. The components are scaled in such a way that the peak ground acceleration in the horizontal direction is $0.4g$, as shown in Fig. 3.

Table 1. Natural Sloshing Periods and Convective Mass Ratios of Shallow and Tall Tank Models

	Natural sloshing periods		Convective mass ratios	
	FEM	Analytical	FEM	Analytical
Shallow tank	8.58	8.56	0.75	0.75
Tall tank	5.24	5.15	0.42	0.44

Modal Analysis

Prior to conducting the time-history analyses, the natural periods of sloshing as well as convective mass ratios are calculated using both finite-element and analytical method for both shallow and tall tanks. The analytical natural frequencies are obtained using Eq. (11). The convective mass ratio is calculated using the following equation according to Housner (1957)

$$\frac{W_c}{W_L} = \frac{1}{3}\sqrt{\frac{5}{2}}\left(\frac{a}{2H_L}\right)\tanh\left[\sqrt{\frac{5}{2}}\left(\frac{2H_L}{a}\right)\right] \tag{12}$$

A comparison between these results is shown in Table 1 which indicates that the FE results are in agreement with analytical values. For shallow tank model the natural frequencies are obtained 8.58 and 8.56 s using the FE method and the analytical method, respectively, while these values are 5.24 and 5.15 s for the tall tank model. The mass ratios also follow a similar trend in which good agreement is achieved. Further details on the calculation of sloshing frequencies and masses are given by Patkas and Karamanos (2007) and Karamanos et al. (2006).

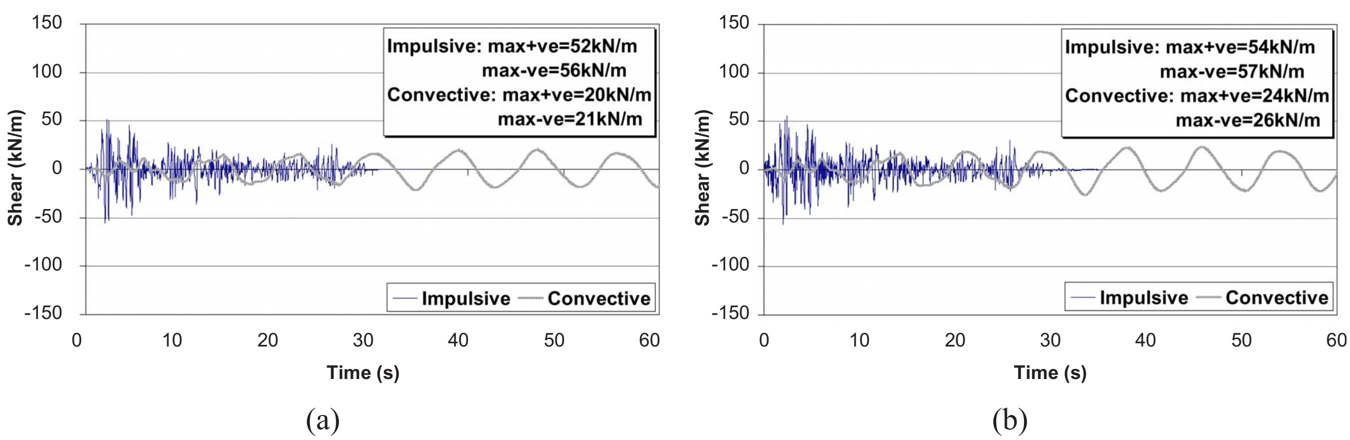


Fig. 4. Time history of base shear for shallow tank model with rigid side walls: (a) horizontal excitation; (b) horizontal and vertical excitation

Table 2. Summary of Dynamic Responses of Shallow Tank and Tall Tank Models

			Impulsive response			Convective response			Combined response		
			H	$H+V$	$(H+V)/H$	H	$H+V$	$(H+V)/H$	H	$H+V$	$(H+V)/H$
Shallow tank	Rigid	Base shear (kN/m)	56	57	1.02	21	26	1.24	63	64	1.01
		Base moment (kN·m/m)	143	144	1.01	58	71	1.22	160	163	1.02
		Sloshing (mm)	—	—	—	527	548	1.04	—	—	—
	Flexible	Base shear (kN/m)	79	83	1.05	21	26	1.24	82	91	1.11
		Base moment (kN·m/m)	257	262	1.02	59	72	1.22	265	284	1.07
		Sloshing (mm)	—	—	—	528	548	1.04	—	—	—
Tall Tank	Rigid	Base shear (kN/m)	176	180	1.02	39	43	1.10	204	203	1.00
		Base moment (kN·m/m)	942	958	1.02	205	223	1.09	1066	1,065	1.00
		Sloshing (mm)	—	—	—	636	708	1.11	—	—	—
	Flexible	Base shear (kN/m)	324	328	1.01	39	44	1.13	323	331	1.02
		Base moment (kN·m/m)	2,283	2,297	1.01	202	218	1.08	2,265	2,275	1.00
		Sloshing (mm)	—	—	—	623	706	1.13	—	—	—

Analyses of Rectangular Tanks

Two rectangular concrete liquid container models given in Fig. 2 are used basically for the example analyses in 2D space. The cross section parallel to short side wall is adopted for 2D FE models including both impulsive and convective responses. To investigate the effect of wall flexibility on dynamic behavior of liquid tanks, both flexible and rigid tank responses have been obtained. Finally, seismic analyses are performed using both horizontal and vertical components of ground acceleration and the results are compared with other relevant methods available in literature.

Behavior of Liquid Tanks with Rigid Wall Boundary Condition

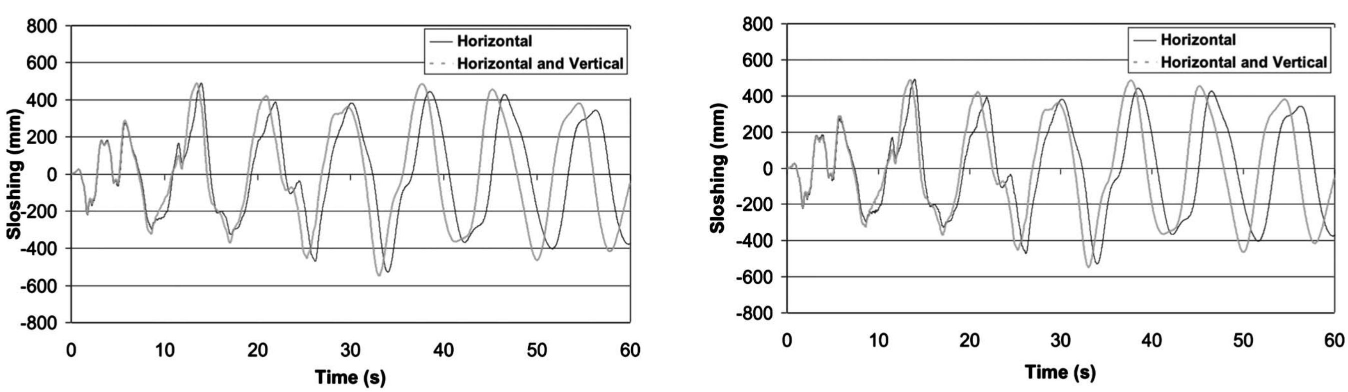
Response of Shallow Tank

The transient base shear and base moment for rigid shallow tank model due to horizontal and vertical excitations acting per unit width of the side wall are calculated by proposed method. The base shear diagram is shown in Fig. 4.

The absolute maximum values of the impulsive response resulting base shear and base moment due to the horizontal excitations are 56 kN/m and 143 kN·m/m, respectively. In this case, the absolute peak values of convective base shear and base moment responses are 21 kN/m and 58 kN·m/m which have occurred around 30 s after peaks of impulsive responses. On the other hand, when the impulsive response reaches its peak, the convective response is at the beginning stage and has not yet fully developed. The maximum absolute values for base shear and base moment due to combined impulsive and convective responses are 63 kN/m and 160 kN·m/m, respectively, which are about 12% higher than those values related to impulsive term.

Considering the combined effect of vertical and horizontal ground motions, the impulsive response almost remains unchanged while the absolute maximum values of convective base shear and base moment increase by around 24 and 22%, respectively. However, the absolute maximum values of base shear and base moment related to combined impulsive and convective terms are slightly increased to 64 kN/m and 163 kN·m/m, respectively, due to vertical excitation effect. A summary of the results are provided in Table 2.

The time-history diagram of surface sloshing height at the top corner of the liquid domain at wall location is shown in Fig. 5(a). Due to effect of vertical acceleration, the absolute maximum values of sloshing has increased from 527 mm at t=33.98 s to 548 mm at t=33.04 s which represents about 4%.



(a) (b)

Fig. 5. Time history of sloshing height at top right corner of fluid domain for shallow tank model: (a) rigid; (b) flexible

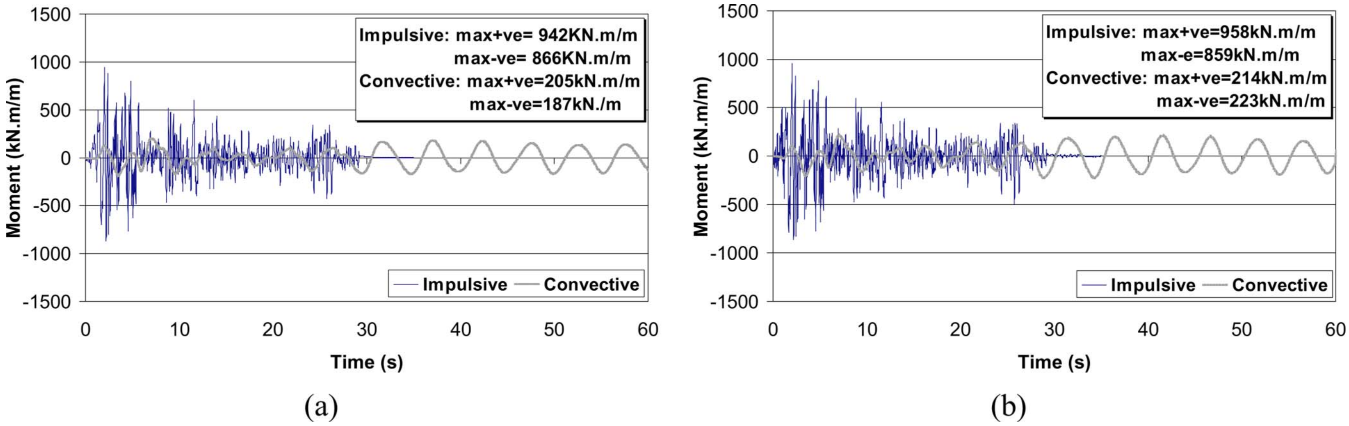


Fig. 6. Time history of base moment for tall tank model with rigid side walls: (a) horizontal; (b) horizontal and vertical

Response of Tall Tank

Fig. 6 presents a diagram of structural response in terms of base moment time history for rigid tall tank model. The absolute maximum values of the resulting base shear and base moment of the impulsive component due to the horizontal excitations are 176 kN/m and 942 kN·m/m, respectively. Considering the free surface motion, the obtained absolute maximum values of convective base shear and base moment responses are 39 kN/m and 205 kN·m/m, respectively, which have occurred at around 5 s following peaks of the impulsive responses. In this case, the absolute maximum values of convective responses are about 20% of those values associated with impulsive responses. However, this ratio was found to be about 40% in shallow tank model. As a result, the convective term is more pronounced in the shallow tank than the tall tank. The maximum absolute values for base shear and base moment due to combined impulsive and convective responses are 204 kN/m and 1,066 kN·m/m, respectively, which are about 15% higher than those values related to impulsive term.

Due to effect of vertical ground motion, the impulsive and convective responses increase by about 2 and 10%, respectively, which shows that the convective term is less sensitive to vertical excitations in the tall tank as compared to the shallow tank model. In this case, the absolute maximum values of base shear and base moment related to combined impulsive and convective terms almost remain unchanged. In addition, the dynamic pressure distribution along the side walls indicates that the effect of vertical ground motion is negligible.

The time-history diagram of sloshing height at the top corner of the liquid domain at wall location is shown in Fig. 7(a). Due to effect of vertical acceleration, the absolute maximum value of sloshing has increased from 636 mm at $t=28.74$ s to 708 mm at $t=28.2$ s. Clearly, this increase due to vertical earthquake component is more than 11% which is higher than its value in shallow tank model.

Behavior of Liquid Tanks with Flexible Walls

Response of Shallow Tank

As mentioned before, to consider the effects of flexibility of tank wall on both dynamic pressure distribution and dynamic response of tank structure, an additional FE flexible wall boundary condition is investigated in this study. The dynamic response of shallow tank structure in terms of base shear is presented in Fig. 8.

Under horizontal excitation, the absolute maximum values of the resulting base shear and base moment are 79 kN/m and 257 kN·m/m as impulsive terms and 21 kN/m and 59 kN·m/m as convective terms, respectively. The corresponding values asso-

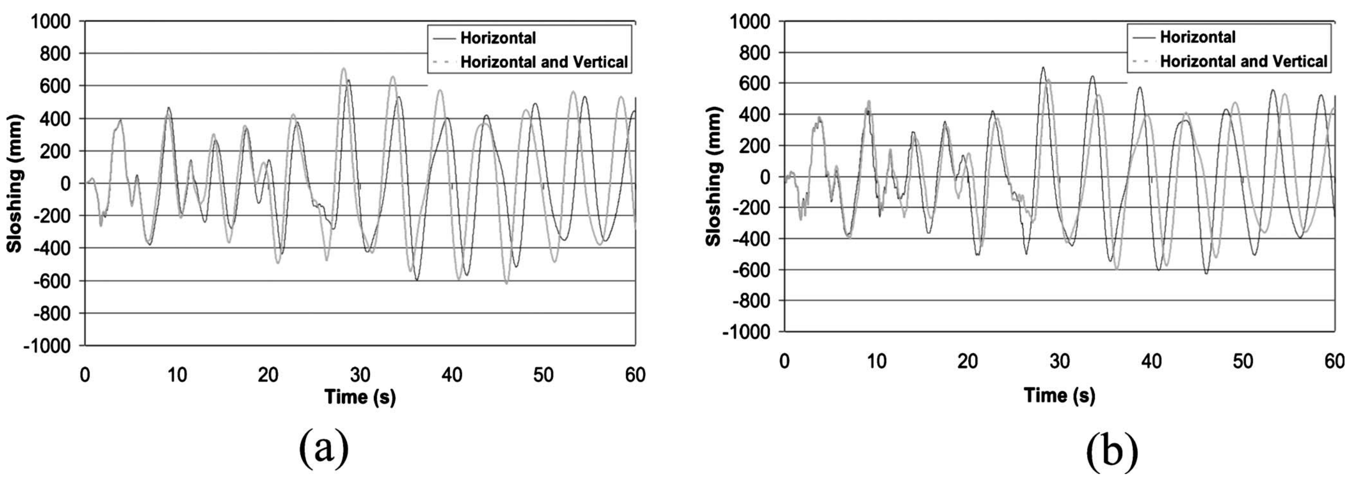


Fig. 7. Time history of sloshing height at top right corner of fluid domain for tall tank model: (a) rigid; (b) flexible

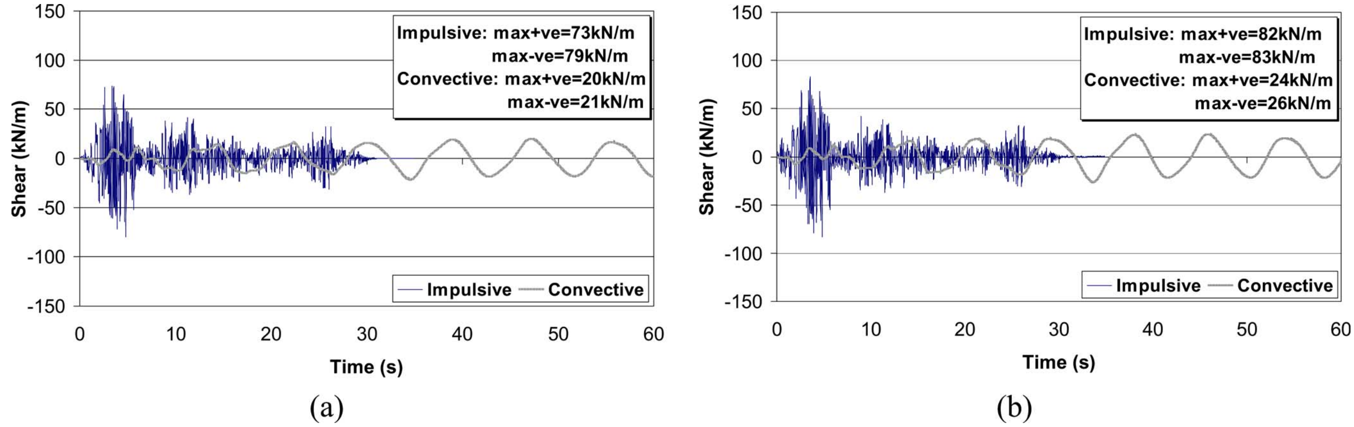


Fig. 8. Time history of base shear for shallow tank model with flexible side walls: (a) horizontal; (b) horizontal and vertical

ciated with combined impulsive and convective are 82 kN/m and 265 kN·m/m for base shear and base moment, respectively. This shows an increase of about 3% due to free surface motion effects which is much lower than the rigid wall model. In comparison with rigid boundary condition results, the maximum values associated with impulsive base shear and base moments are significantly increased by about 40 and 80% due to flexibility of side walls, respectively. However, the effect of wall flexibility on convective response is negligible. A summary of the results is provided in Table 2.

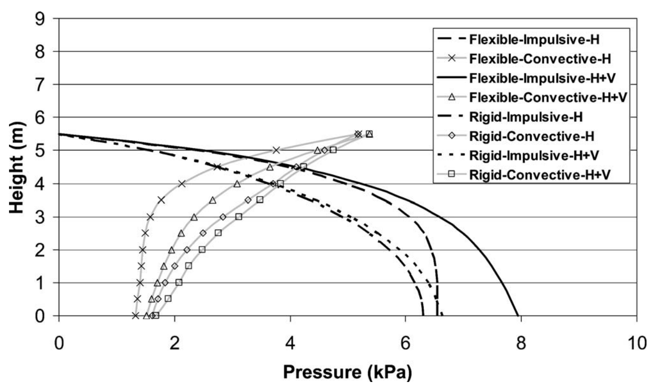


Fig. 9. Impulsive and convective pressure distribution along height of shallow tank wall for both rigid and flexible wall conditions

Similar to the rigid tank, the vertical component of earthquake affects the convective term more than that of impulsive. The convective base shear and base moment maximum values have increased by 24 and 22%, respectively. The increase for impulsive terms is about 2%.

The pressure distribution along the tank height is shown in Fig. 9. In fact, hydrodynamic pressure tends to be amplified due to fluid-structure interaction effect in a flexible container and its distribution differs from that in the corresponding rigid container. However, the effect of wall flexibility will lead to a slight reduction of about 2% in surface convective pressure values as shown in this diagram. Although most current codes and standards assume that the value of convective pressure at the bottom of tank is 0, the obtained results show different trend for both rigid and flexible models.

The time-history diagram of sloshing height at the top corner of the liquid domain at wall location is shown in Fig. 5(b). Due to effect of vertical acceleration, the absolute maximum values of sloshing has increased from 528 mm at $t=34$ s to 548 mm at $t=33$ s which are in agreement with those values obtained using rigid boundary condition.

Response of Tall Tank

Fig. 10 shows the structural response in terms of base moment time history for flexible tall tank model. In comparison with the rigid boundary condition, the maximum absolute values of impul-

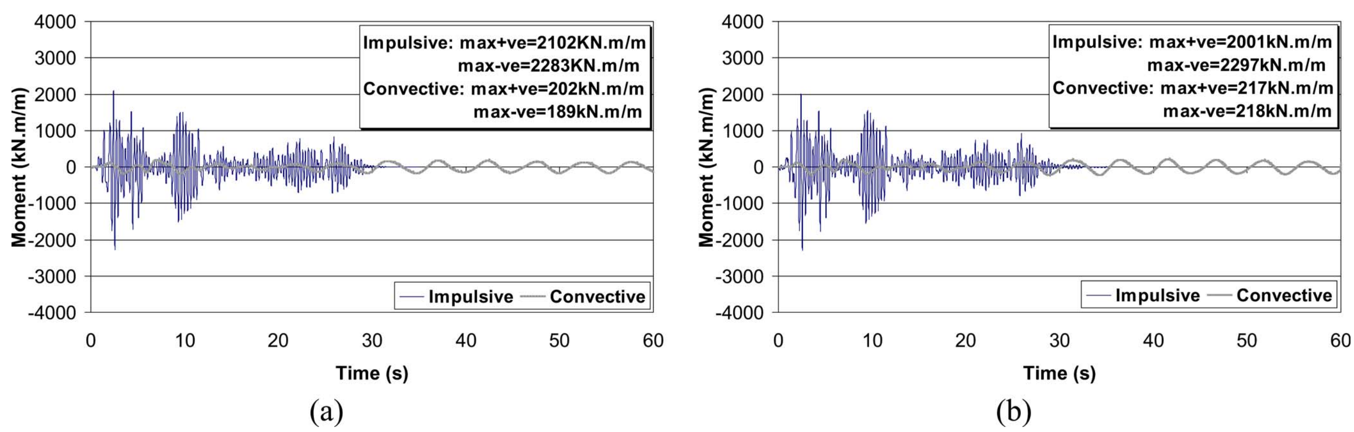


Fig. 10. Time history of base moment for tall tank model with flexible side walls: (a) horizontal; (b) horizontal and convective

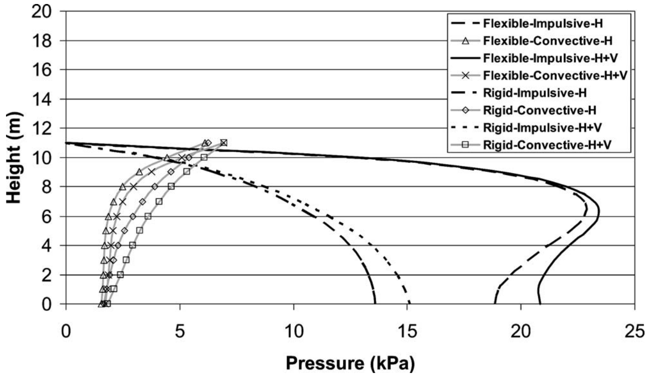


Fig. 11. Impulsive and convective pressure distributions along height of flexible side wall of tall tank model for both rigid and flexible wall conditions

sive base shear and base moment due to horizontal earthquake component are increased by about 85 and 140% to the values of 324 kN/m and 2,283 kN·m/m, respectively. Although the effect of flexibility on impulsive terms is more significant in tall tank model than shallow tank model, the effect on convective component is insignificant as indicated in Table 2.

Accounting for the vertical acceleration effects, the convective terms show an average increase of about 10% while of the impulsive term the amount of increase is only about 1%. For combined impulsive and convective terms, the maximum absolute values of base shear and base moment are 323 kN/m and 2,265 kN·m/m due to horizontal ground motion and 331 kN/m and 2,275 kN·m/m due to both horizontal and vertical earthquake components. In this case, it can be seen that the free surface motion leads to a slight decrease in the overall structural response of the wall when compared to the response due to impulsive component only. This phenomenon can be explained as the phase difference between impulsive and convective responses which depends on various parameters such as tank configuration, earthquake frequency, and fluid-structure boundary conditions.

The pressure distribution curves for tall tank model presented in Fig. 11 show that hydrodynamic pressure is significantly amplified in the middle of the flexible side wall due to wall flexibility. It seems that amplification of dynamic pressure is more pronounced in tall tank model than shallow tank model. Similar to shallow tank models, convective pressure values decrease due to wall flexibility as shown in this figure.

Fig. 7(b) shows the time history of the sloshing height for this case recorded at the top corner of the fluid domain at wall location. The maximum sloshing heights due to horizontal and combined horizontal and vertical excitation are 623 mm at t=28.8 s and 706 mm at t=28.2 s, respectively. The vertical component of earthquake results in an increase in sloshing height of about 13%. The increase in sloshing heights due to the effect of vertical acceleration is higher in tall tank model than shallow tank model and should not be neglected. In addition, the wall flexibility does not have a noticeable effect on sloshing heights as a similar trend was also observed for the shallow tank.

Results Summary and Comparison with Other Methods

The seismic impulsive and convective responses of liquid-tank models are obtained in this study as discussed previously considering both flexible and rigid boundary conditions and fluid damping properties. In current analytical methods and lumped mass approximations the impulsive wave absorbance boundary condition and fluid viscosity are ignored. To verify the proposed FE method as well as to consider the effect of fluid damping properties, two different conditions with zero and nonzero fluid damping ratios are used for rigid tank models and the results are compared with analytical solutions. These results are presented in terms of impulsive hydrodynamic pressure over the tank height.

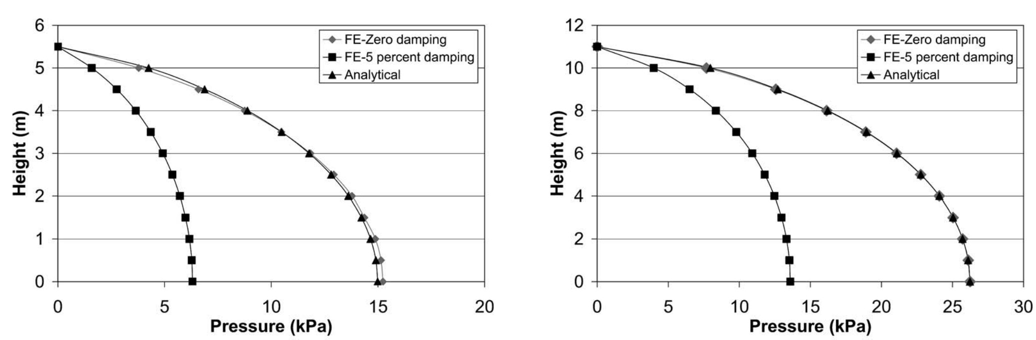
The analytical impulsive pressure distribution for the rigid wall is given as following equation which is the same that derived by Haroun and Housner (1981) in reference:

$$P = \sum_{n=1}^{\infty} \frac{2(-1)^n \rho_l}{\lambda_{i,n}^2 \, . \, H_l} \tanh(\lambda_{i,n} L_x) \cos(\lambda_{i,n} y) \ddot{u}_g(t) \qquad (13)$$

In which $\lambda_{i,n}=[(2n-1)\pi]/2H_l$ and $\ddot{u}_g$ is the horizontal ground acceleration.

The impulsive hydrodynamic pressure distributions over the rigid tank wall calculated by both Eq. (13) and FEM with two different damping conditions are shown in Fig. 12 for both shallow and tall tanks. It is obvious that the FE pressure distribution is in full agreement with analytical results when the fluid damping is ignored. For a 5% damping ratio, the impulsive pressure has decreased by almost 50% for both shallow and tall tank models. This indicates that the fluid damping is an important parameter in considering the seismic behavior of liquid containers. This fact is not directly considered in current codes and standards which are based on lumped mass models.

A summary of FE results is presented in Table 2 for both shallow and tall tank models. The base shear and base moment



(a) (b)

Fig. 12. Impulsive hydrodynamic pressure distribution over rigid wall tank: (a) shallow tank; (b) tall tank

responses are very similar to those reported by Kianoush et al. (2006) for shallow tank model under scaled components of El-Centro earthquake with peak acceleration of $0.4g$. It should be noted that the tank analyzed by Kianoush et al. (2006) was slightly different in dimensions from that used in the current study.

The same tall tank model was investigated by Kim et al. (1996) using a BEM-FEM under horizontal excitation of El-Centro record with peak acceleration of $0.32g$. The reported base shear response is almost 60% higher than the corresponding value in current study. This anomaly was also observed in the study by Chen and Kianoush (2005).

The impulsive responses are also comparable to those obtained by Chen and Kianoush (2005) using a sequential method under horizontal component of El-Centro earthquake with peak ground acceleration of $0.32g$ for both shallow and tall tank models. In their sequential method, the flexibility of wall was taken into account. However, only the impulsive behavior of tank was considered and the convective component was ignored.

In their study, the maximum impulsive base shear and base moments are 78.7 kN/m and 241.8 kN·m/m for shallow tank model while the maximum impulsive base shear is 338.1 kN·m/m for tall tank model. These values are similar to the proposed FE results. However, it is clear that for a peak ground acceleration of $0.4g$ used in this study slightly higher values are expected using the sequential method.

In addition, the dynamic responses are calculated by Housner's method in which the mapped spectral response acceleration S_s and S_1 for Imperial Valley location are chosen as $1.5g$ and $0.6g$, respectively. It should be noticed that in this approximation the flexibility of wall and fluid damping are ignored.

According to Housner's method, for shallow tank model, the impulsive base shear and base moments are 165 kN/m and 366 kN·m/m under horizontal excitations while for tall tank model, the impulsive base shear and base moments are 689 kN/m and 3,199 kN·m/m, respectively. These values are much higher than those obtained using FE method. However, the FE convective response is in satisfactory agreement with Housner's model in terms of base shear and sloshing height for rigid wall boundary condition. For instance, the convective base shear calculated by Housner's method for tall tank model under horizontal ground motion is 34 kN/m which is only 13% less than the value obtained using FE method. For this case, the sloshing height is 710 mm based on Housner's model which is about 10% higher than the value obtained using the FE method.

The proposed FE results show that considering wall flexibility in fluid-structure interaction will amplify the dynamic response of the system which is ignored in current design practice. This effect is more important in impulsive than convective response. The effect of wall flexibility results in minor changes in convective response which indicates that the convective behavior is almost independent of the flexibility characteristics of the side walls. This phenomenon may be a result of the nature of free motion boundary condition which is based on the gravity wave theory.

Conclusions

In this study, a FEM is introduced that can be used for the analysis of dynamic behavior of partially filled rectangular fluid container under horizontal and vertical ground excitations. The liquid sloshing is modeled using an appropriate boundary condition and the damping effects due to impulsive and convective components of the stored liquid are modeled using the Rayleigh method. Two different configurations including shallow and tall tank models are considered to investigate the effect of geometry on the response of the liquid-structure system. Effect of wall flexibility on the overall dynamic response of system is investigated by comparing the results between rigid and flexible models.

The results show that the maximum impulsive base shear and base moment obtained from time-history analysis of the considered system are increased due to flexibility of side walls which is a result of dynamic pressure amplification in the middle of the wall. However, the convective response is almost independent of variations of flexibility of the side walls and seems to be related to geometric configurations of tank, earthquake characteristics, and liquid properties. Due to wall flexibility, a slight reduction is observed in convective pressure values. Although the current practice assumes that the numerical value of convective pressure at the bottom of tank reaches to 0, the proposed FE method shows different results.

Furthermore, the peak responses of impulsive and convective components do not occur at the same phase and time. As a result, convective terms may lead to increase or decrease the maximum absolute values of the structural responses in terms of base shear and base moment. Also, applying the vertical excitations will lead to an increase in the convective response of the system. However, it does not affect the impulsive behavior significantly. This increase is more noticeable in tall tank model.

Although the FE convective responses are in satisfactory agreement with corresponding responses obtained by Housner's method, the seismic impulsive response of liquid tanks according to Housner's method which is used currently in current codes and standards in terms of shear and moment forces seems too conservative.

It is clear that the dynamic behavior of liquid concrete tanks depends on a wide range of parameters such as seismic properties of earthquake, tank dimensions and fluid-structure interaction which should be considered in current codes of practice. This study shows that the proposed FE method can be used in the time-history analysis of rectangular liquid tanks. One of the major advantages of this method is in accounting for damping properties of liquid domain and calculating impulsive and convective terms separately. Also, this includes applying different damping ratios to different impulsive and convective components and considering wall flexibility which has a great role in seismic response.

The present study was done based on 2D analysis of rectangular tanks. As a suggestion for future study in this field, such a 2D analysis can be extended to three-dimensional analysis which would make the problem more interesting, challenging, and realistic.

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